

National Center on
INTENSIVE INTERVENTION

at American Institutes for Research ■

Fractions as Numbers

While permission to reprint this publication is not necessary, the citation should be:

National Center on Intensive Intervention. (2014). Fractions as Numbers. Washington, DC: U.S. Department of Education, Office of Special Education Programs, National Center on Intensive Intervention.

This document was produced under the U.S. Department of Education, Office of Special Education Programs, Award No. H326Q110005. Celia Rosenquist serves as the project officer. The views expressed herein do not necessarily represent the positions or policies of the U.S. Department of Education. No official endorsement by the U.S. Department of Education of any product, commodity, service or enterprise mentioned in this website is intended or should be inferred.

Contents

1. Fractions as Numbers: Considerations for Instruction	5
2. Fraction Equivalence	11
Sample Fraction Equivalence Activities (1–4)	12
a. Activity One: Using Fraction Tiles and Fraction Circles	12
b. Activity Two: Matching Equivalent Fractions	15
c. Activity Three: Matching Equivalent Fractions	17
d. Activity Four: Fluency Building With Equivalent Fractions	19
Worksheet	21
a. Fraction Equivalence	21
b. Identifying Equivalent Fractions	24
c. Making Equivalent Fractions	28
d. Finding Equivalent Fractions #1	30
e. Finding Equivalent Fractions #2	32
3. Fraction Magnitude	34
Sample Fraction Magnitude Activities (1–2)	35
a. Activity One: Comparing Fractions With Different Denominators	35
b. Activity Two: Comparing Fractions With Different Denominators	40
Worksheet	43
a. Fraction Magnitude: Comparing Fractions With Different Denominators	43
b. Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators	45

4.	Converting Between Mixed Numbers and Improper Fractions48
	Sample Activity49
	a. Converting Between Mixed Numbers and Improper Fractions49
	Worksheet54
	a. Converting Mixed Numbers and Improper Fractions54
5.	Supplemental Materials (used across activities)57
	a. Fraction Equivalence Tiles58
	b. Fraction Circles59
	c. Fraction Equivalence Cards With Images61
	d. Fraction Equivalence Cards Without Images65
	e. Best Time Score Card68
	f. Fraction Comparison Flash Cards69
	g. Multiplication Chart73
	h. Improper Fraction Flash Cards74
	i. Mixed-Number Flash Cards80
	j. Number Line85

Fractions as Numbers: Considerations for Instruction

Purpose and Overview of Guide

The purpose of this guide and companion materials is to support developing and implementing lessons for students who need intensive instruction in the area of understanding fractions as numbers. Special education teachers, mathematics interventionists, and others working with students struggling in the area of fractions may find this guide helpful. Additional sample activities, worksheets, and supplemental materials are also available for download on the NCII website.

Within college- and career-ready standards, fractions are taught in Grades 3–5. This guide may be used as these concepts are introduced or with students in higher grade levels who continue to struggle with the concepts.

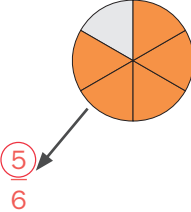
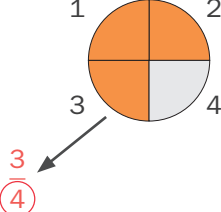
Sequence of Skills—College- and Career-Ready Standards

Develop an understanding of fractions as numbers:

- Part/whole relationship
- Number on the number line
- Equivalent fractions
- Whole numbers as fractions
- Comparing fractions

Language/Symbols

The following terms are important for students to understand when working with fractions.

<p>Fraction: A part of a whole, with all parts equivalent.</p> <p>$\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{1}{8}, \frac{4}{5}$</p>	<p>Numerator: How many parts of the whole.</p> 	<p>Denominator: How many parts make up the whole.</p> 
<p>Common Denominator: One or more fractions have the same denominator. Necessary for adding and subtracting fractions.</p> <p>$\frac{1}{8} + \frac{2}{8}$</p>	<p>Equivalent Fractions: Fractions with equal value.</p> <p>$\frac{4}{6} = \frac{2}{3} \quad \frac{2}{8} = \frac{1}{4}$</p>	<p>Unit Fraction: A fraction with 1 in the numerator.</p> <p>$\frac{1}{12}, \frac{1}{8}, \frac{1}{5}, \frac{1}{3}, \frac{1}{2}$</p>

Conceptual Understanding

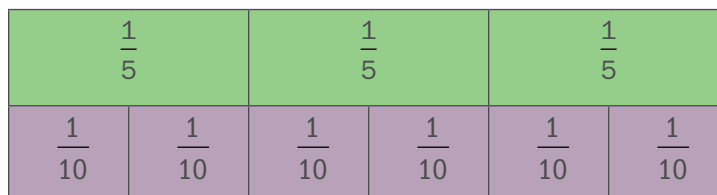
Fraction tiles, fraction circles, or other manipulatives can be used to help students visualize and conceptually understand many fraction concepts. These manipulatives should represent 1 whole, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$.

Develop understanding of fractions as numbers, such as the following:

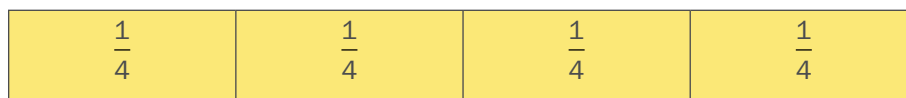
$\frac{1}{5}$ is the same as $\frac{2}{10}$.

$\frac{2}{5}$ is the same as $\frac{4}{10}$.

$\frac{3}{5}$ is the same as $\frac{6}{10}$.



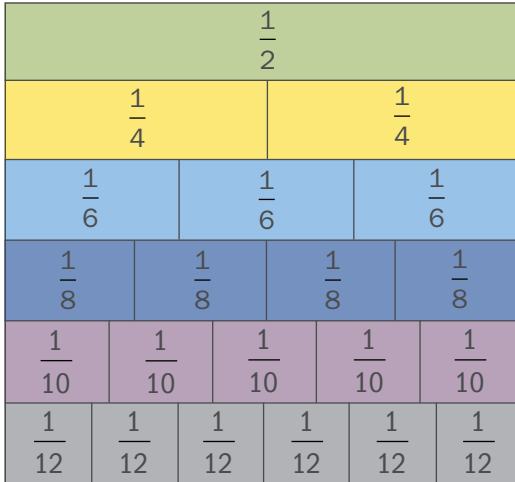
$\frac{4}{4}$ is the same as 1.



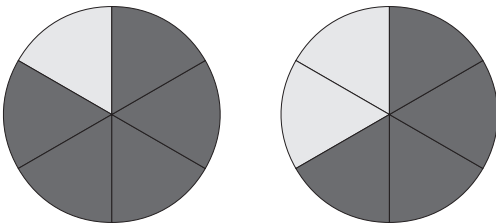


1

$$1/2 = 2/4 = 3/6 = 4/8 = 5/10 = 6/12$$

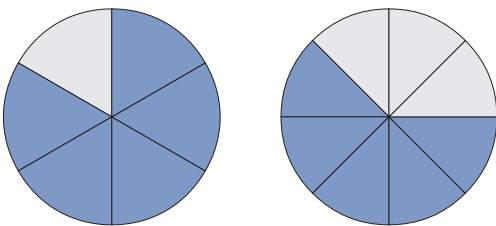


Comparing fractions with like denominators:



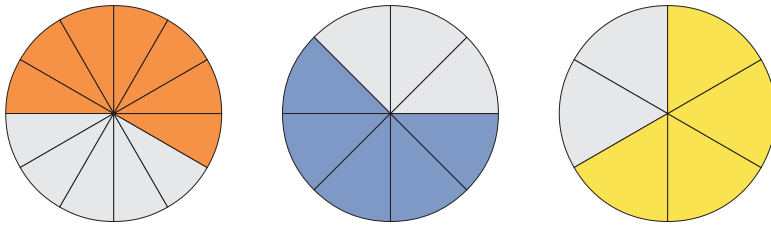
By showing both $5/6$ and $4/6$ using fraction circles, students can see that $5/6 > 4/6$.

Comparing fractions with the same numerator but unlike denominators:



By showing both $5/6$ and $5/8$ using fraction circles, students can see that $5/6 > 5/8$. This will help them understand that although 8 is larger than 6, sixths are larger than eighths in fractions.

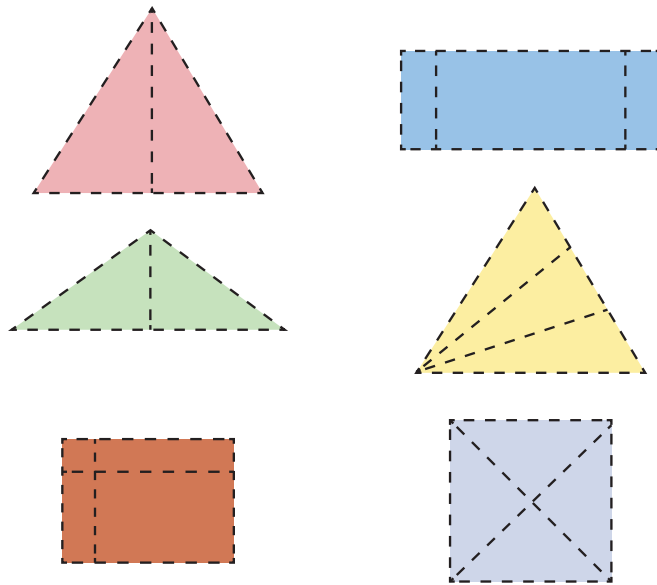
Putting fractions in descending order:



By showing $7/12$, $5/8$, and $4/6$ using fraction circles, students see that $7/12 < 5/8 < 4/6$.

Determine if parts are equal:

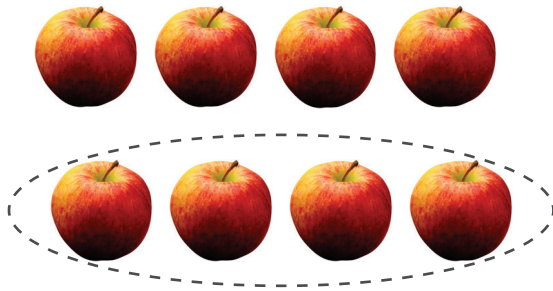
Students must understand that fractions are equal parts of a whole. If students have difficulty with the concept of equal, provide them with two-dimensional pictures and have them identify which ones are divided into equal parts and which ones are not. This will allow teachers to assess a student's knowledge of this concept.



Understanding part of a group:

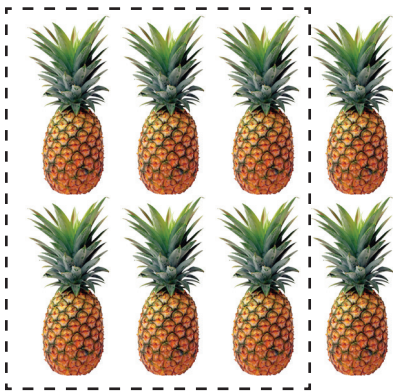
Understanding that fractions can be part of a group, or set, is important because students will often hear fractions being used to describe objects.

Circle $\frac{1}{2}$ of the apples.



Students must first understand that the apples need to be in two equal groups, as the denominator states. Students should realize that the two rows are equal and would circle one row.

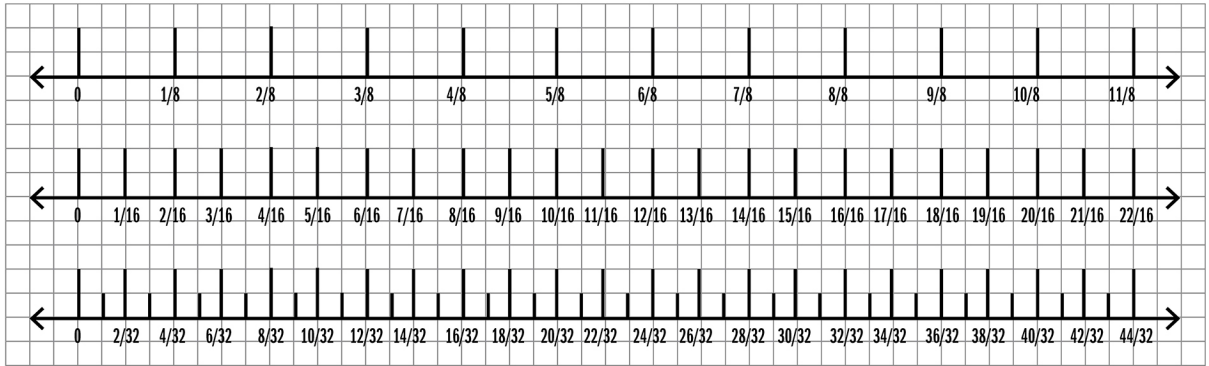
How many are $\frac{3}{4}$ of the pineapples?



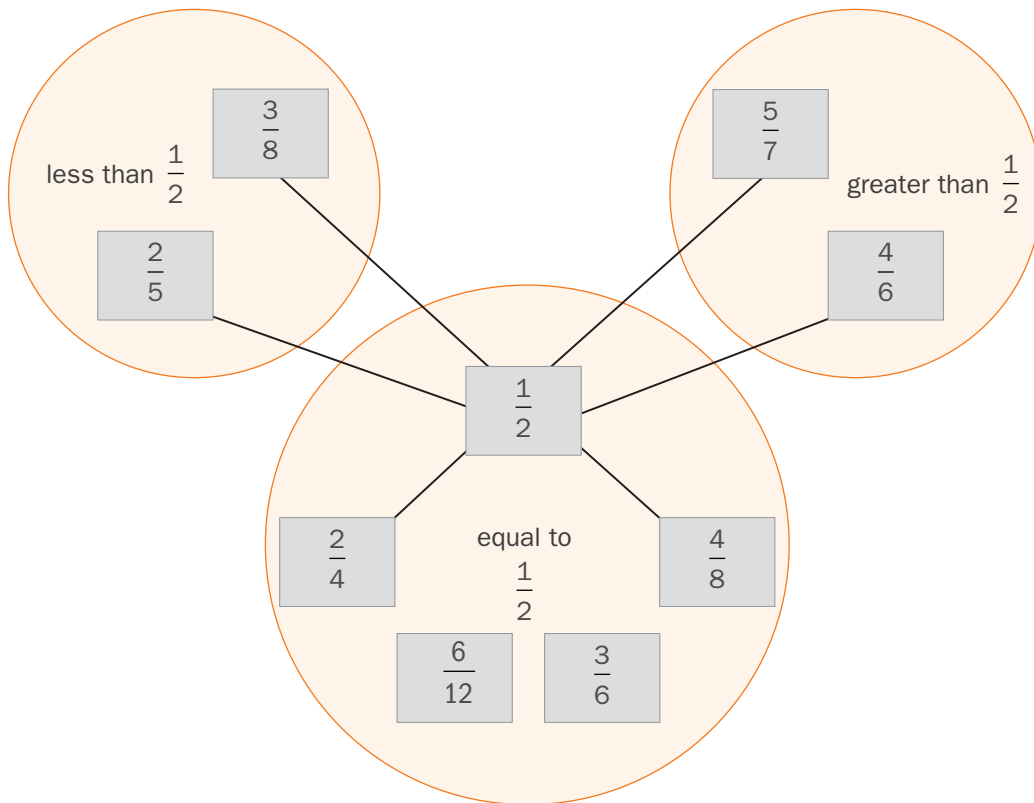
Students should realize that there are four columns (which would be the denominator), so they should circle three columns (or three of the four groups). This will show them that $\frac{3}{4}$ of the 8 pineapples is 6 pineapples.

Number lines can be used to help students understand the relationship between whole numbers and fractions.

- Show students that $\frac{1}{8} = \frac{2}{16} = \frac{4}{32}$.
- Show students that $\frac{4}{16} < \frac{1}{8}$.
- Show students that $\frac{8}{8} = 1$, $\frac{16}{16} = 1$, and $\frac{32}{32} = 1$.



Graphic organizers may help students understand concepts by visually organizing concepts. The following example shows fractions that are less than, equal to, and greater than $\frac{1}{2}$. Teachers could give this graphic organizer to students in blank form, give them fractions, and have them put the fractions in the correct spots. The graphic organizer also could be partially completed prior to having students interact with it.



2. Fraction Equivalence

Sample Activities 1-4

- a. Activity One: Using Fraction Tiles and Fraction Circles**
- b. Activity Two: Matching Equivalent Fractions**
- c. Activity Three: Matching Equivalent Fractions**
- d. Activity Four: Fluency Building With Equivalent Fractions**

Worksheets

- a. Fraction Equivalence**
- b. Identifying Equivalent Fractions**
- c. Making Equivalent Fractions**
- d. Finding Equivalent Fractions #1**
- e. Finding Equivalent Fractions #2**

Sample Fraction Equivalence Activities (1–4)

College- and Career-Ready Standards Addressed:

3.NF.3. Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

- Understand two fractions as equivalent (equal) if they are the same size or occupy the same point on the number line.
- Recognize and generate simple equivalent fractions (e.g., $1/2 = 2/4$, $4/6 = 2/3$). Explain why the fractions are equivalent (e.g., by using a visual fraction model).

Activity One: Using Fraction Tiles and Fraction Circles

Purpose: Identify fractions equivalent to $1/2$.

Principles of Intensive Intervention Illustrated:

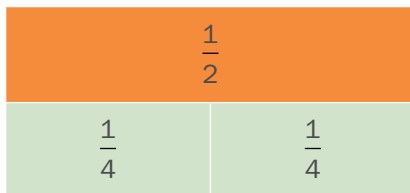
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide concrete learning opportunities (including use of manipulatives).
- Provide repeated opportunities to correctly practice skills.
- Provide feedback and explicit error correction. Have the student repeat the correct process when he or she makes an error.

Materials (available for download from NCII):

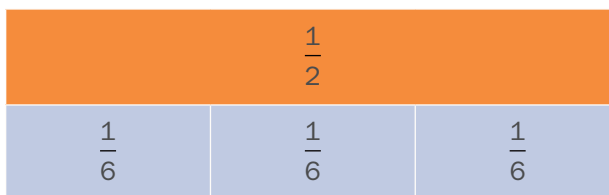
- Fraction tiles or fraction circles (see Supplemental Materials Section)
- Worksheet: Fraction Equivalence (for extra practice)

Modeling:

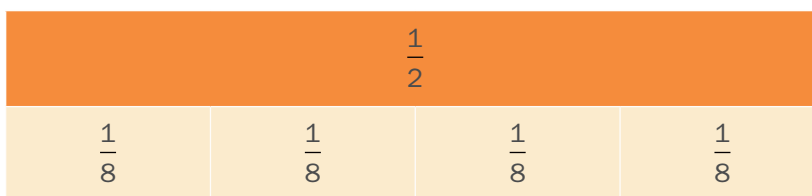
1. Place the $\frac{1}{2}$ fraction bar in front of the student.
2. Place two $\frac{1}{4}$ fraction bars under the $\frac{1}{2}$ bar.



3. Explain that because $\frac{1}{2}$ and two $\frac{1}{4}$ bars are the same size, $\frac{1}{2}$ is equal to $\frac{2}{4}$.
4. Write $\frac{1}{2} = \frac{2}{4}$.
5. Provide one more example with $\frac{1}{2}$ and three $\frac{1}{6}$ bars.

**Guided Practice:**

1. Tell the student to take out the $\frac{1}{2}$ bar.
2. Tell the student to take out the $\frac{1}{8}$ bars.
3. Tell the student to see how many $\frac{1}{8}$ bars it takes to be equal with the $\frac{1}{2}$ bar.
4. Student response: 4.

**Feedback:**

5. If correct, say, "Yes, $\frac{4}{8}$. You can see $\frac{1}{2}$ is the same as $\frac{4}{8}$. They're the same size. That means they're equivalent."
6. Have the student write the equivalent fraction: $\frac{1}{2} = \frac{4}{8}$.

Practice:

7. Repeat with $\frac{5}{10}$ and $\frac{6}{12}$.

Note: The tiles and circles can be used to show equivalence for the following:

$$\frac{1}{4} = \frac{2}{8} \text{ and } \frac{3}{12}$$

$$\frac{1}{5} = \frac{2}{10}$$

$$\frac{1}{6} = \frac{2}{12}$$

Corrective Feedback:

Sample incorrect student response 1: “ $1/2$ is equivalent to $3/8$.”

Teacher feedback: “ $3/8$ is not quite enough. Look at the tiles; $3/8$ is shorter than $1/2$. The fractions have to be the same size to be equivalent. Try it again. How many eighths are equivalent to $1/2$?”

Sample incorrect student response 2: “ $1/2$ is equivalent to $5/8$.”

Teacher feedback: Determine why the student made the error. Was it a counting error? Was it a visual/spatial error?

“ $5/8$ is too much. Look at the tiles: $5/8$ is longer than $1/2$. The fractions have to be the same size to be equivalent. Try it again. How many eighths are equivalent to $1/2$?”

(Have the student demonstrate the correct procedure following the error before moving to the next problem.)



Activity Two: Matching Equivalent Fractions

Purpose: To identify equivalent fractions.

Principles of Intensive Intervention Illustrated:

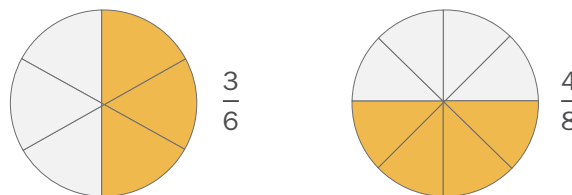
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide concrete learning opportunities (including use of manipulatives).
- Provide repeated opportunities to correctly practice the step.
- Provide feedback and explicit error correction. Have the student repeat the correct process if he or she makes errors.

Materials (available for download from NCII):

- Fraction equivalence circle cards (see Supplemental Materials section)
- Worksheet: Identifying Equivalent Fractions (for extra practice)
- Worksheet: Making Equivalent Fractions (for extra practice)

Modeling:

1. Lay out all cards on the table.
2. Pick one card that shows $\frac{3}{6}$ and say, "This circle shows $\frac{3}{6}$." Count out the total parts (6) and then the colored parts (3), if necessary.
3. Pick a second card that shows $\frac{4}{8}$ and say, "This circle shows $\frac{4}{8}$." Count out total the parts (8) and then the colored parts (4), if necessary.
4. Explain to the students that $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions because they are both $\frac{1}{2}$ of the circle.



Guided Practice:

1. Have the student select one card and name the fraction.
2. Have the student select another card that shows an equivalent fraction.
3. Have the student explain the rationale and provide appropriate positive or corrective feedback.

Corrective Feedback:

Sample incorrect student response: “ $2/4$ is equivalent to $3/5$.”

Teacher feedback: “ $3/5$ is bigger than $2/4$. Look at the picture: $3/5$ is greater than $2/4$. $2/4$ is equivalent to half the circle, but $3/5$ is more than half the circle. Fractions have to be the same size to be equivalent.

Try again. Is there a different card that is equivalent to $2/4$?”

(Have the student demonstrate the correct procedure following the error before moving to the next problem.)



Activity Three: Matching Equivalent Fractions

Purpose: To identify equivalent fractions without the aid of pictures and to teach students the rule for finding equivalent fractions.

Principles of Intensive Intervention:

- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Provide repeated opportunities to correctly practice the step.

Materials (available for download from NCII):

- Fraction equivalence cards without images (see Supplemental Materials section)
- Worksheet: Finding Equivalent Fractions #1

Modeling:

Explain the **Equivalent Fraction Rule:** To find an equivalent fraction, you multiply the numerator and the denominator by the same number. The new fraction is equivalent to the original fraction.

1. Lay out all cards on the table.
2. Pick one card.
3. Pick a second card that has an equivalent fraction.
4. Explain your thinking: “I know $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ because when I multiply the numerator in $\frac{1}{2}$ (point to 1) by 2 AND I multiply the denominator in $\frac{1}{2}$ (point to 2) by 2, the answer is $\frac{2}{4}$. That tells me $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.” (Demonstrate the multiplication on a piece of paper.)
5. Repeat the procedure.

Guided Practice:

1. The student selects one card.
2. The student will select a second card that shows an equivalent fraction.
3. The student will explain or show how he or she knows it is equivalent.
4. The student will show the multiplication procedure (see modeling section earlier) and explain. If the student gets stuck during the explanation, allow the student to create the fractions with fraction circles or tiles.

Corrective Feedback:

Sample incorrect student response: “ $1/2$ is equivalent to $4/6$.”

Teacher feedback: “Let’s check your answer. Look at the rule. (Read the rule: equivalent fraction rule: “To find an equivalent fraction, you multiply the numerator and the denominator by the same number. The new fraction is equivalent to the original fraction.”) Let’s multiply the numerator and the denominator in $1/2$ by the same number to see whether we get $4/6$. Let’s do the numerator first. What can we multiply 1 by to get 4 in the new numerator?”

Student: 4.

“Great. If we multiply the numerator by 4, we have multiply the denominator by 4 to find out whether the fractions are equivalent. Is 2 times 4 equal to 6? No, 2 times 4 equals 8, so we know $4/6$ is NOT equivalent to $1/2$.”

(Have the student demonstrate the correct procedure following the error before moving to the next problem.)

Activity Four: Fluency Building With Equivalent Fractions

Purpose: To identify equivalent fractions without the aid of pictures.

Build quick retrieval of the most common fraction equivalencies ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{3}{4}$)

(Note: This activity does not have to be limited to fractions given. The teacher should determine which fractions to use in this activity.)

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Once the student can complete entire examples and explain his or her work, incorporate fluency-building activities.

Materials (available for download from NCII):

- Fraction equivalence flash cards (without images)
- Best Time Score Card for tracking student progress (see Supplemental Materials section)
- Worksheet: Finding Equivalent Fractions #2 (for extra practice)

Modeling:

Review the **Equivalent Fraction Rule:** To find an equivalent fraction, you multiply the numerator and the denominator by the same number. The new fraction is equivalent to the original fraction.

1. Pick the target fraction ($\frac{1}{2}$ for this example).
2. Tell the student that when a card is flashed, he or she should say whether it is equal or not equal to $\frac{1}{2}$.
3. Give the student 30 seconds to complete as many flash cards as he or she can. (Time can be adjusted to student needs.)
4. The teacher puts cards in correct and incorrect piles.
5. After 30 seconds, the student and the teacher look at the incorrect pile and use manipulatives or the multiplication rule to show why they are or are not equivalent to $\frac{1}{2}$.

6. The student tries to beat his or her score each day to increase quick retrieval and fluency.
7. As the student becomes fluent with one fraction, try a new target fraction.
8. Graph the student's daily progress so that he or she can see improvement.

Corrective Feedback:

Sample incorrect student response: When $4/8$ is flashed, the student says “not equal.”
(The corrective feedback occurs at the end of the 30 seconds.)

Teacher feedback: “Let’s look through the pile of incorrect responses. $4/8$ is in the incorrect pile. Let’s use multiplication to check whether it is equal or not equal to $1/2$. (The teacher demonstrates multiplying as the student answers questions.) Let’s multiply the numerator and the denominator in $1/2$ by the same number to see whether we get $4/8$. Let’s do the numerator first. What can we multiply 1 by to get 4 in the new numerator?”

Student: 4.

Can we multiply 2 by 4 to get 8?

Student: Yes.

Is $4/8$ equal to $1/2$?

Student: Yes.

(Have the student demonstrate the correct procedure following the error before moving to the next problem.)

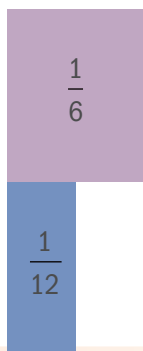
Worksheet

Fraction Equivalence

Objective: Understand fraction equivalence.

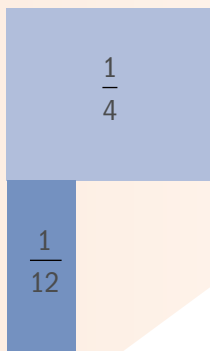
Directions: Show fraction equivalence using fraction tiles.

1. How many $\frac{1}{12}$ bars are equivalent to $\frac{1}{6}$?



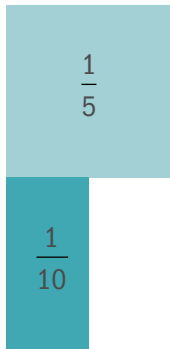
Write an equivalent fraction: _____

2. How many $\frac{1}{12}$ bars are equivalent to $\frac{1}{4}$?



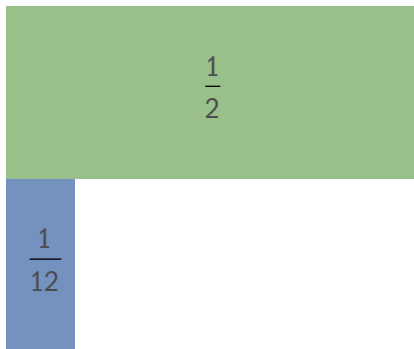
Write an equivalent fraction: _____

3. How many $\frac{1}{10}$ bars are equivalent to $\frac{1}{5}$?



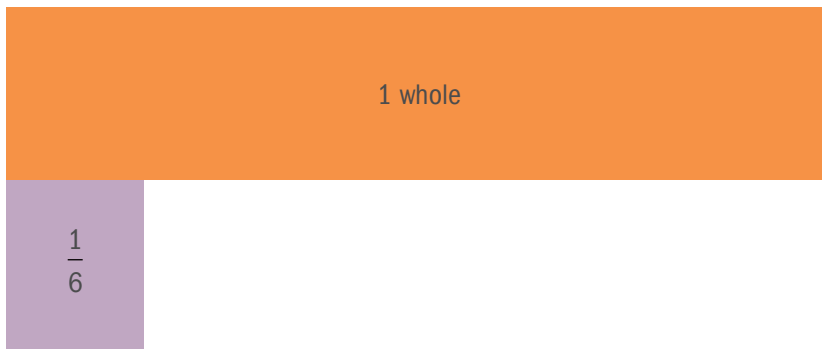
Write an equivalent fraction: _____

4. How many $\frac{1}{12}$ bars are equivalent to $\frac{1}{2}$?



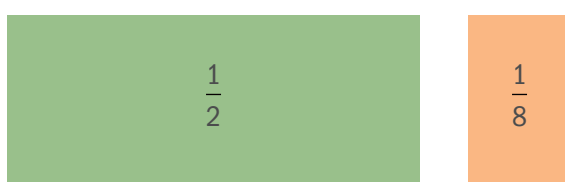
Write an equivalent fraction: _____

5. How many $\frac{1}{6}$ bars are equivalent to 1 whole?



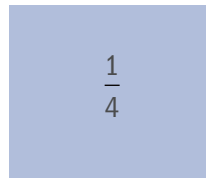
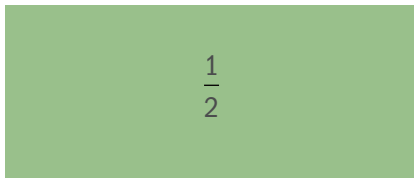
Write an equivalent fraction: _____

6. How many $\frac{1}{8}$ bars are equivalent to $\frac{1}{2}$?



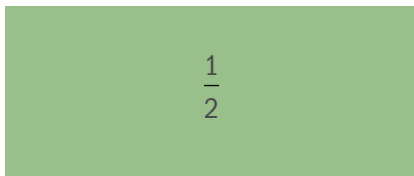
Write an equivalent fraction: _____

7. How many $\frac{1}{4}$ bars are equivalent to $\frac{1}{2}$?



Write an equivalent fraction: _____

8. How many $\frac{1}{6}$ bars are equivalent to $\frac{1}{2}$?



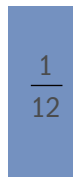
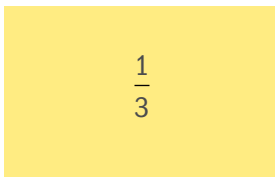
Write an equivalent fraction: _____

9. How many $\frac{1}{6}$ bars are equivalent to $\frac{1}{3}$?



Write an equivalent fraction: _____

10. How many $\frac{1}{6}$ bars are equivalent to 1 whole?



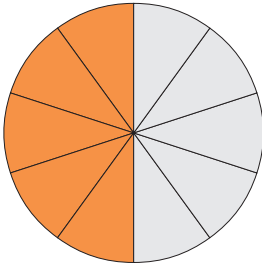
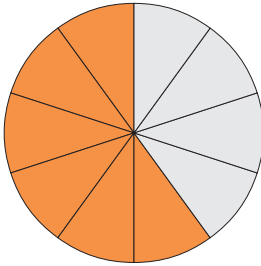
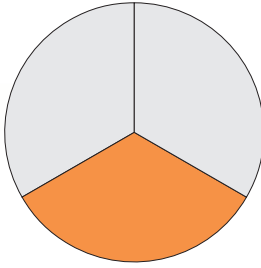
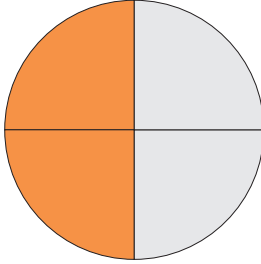
Write an equivalent fraction: _____

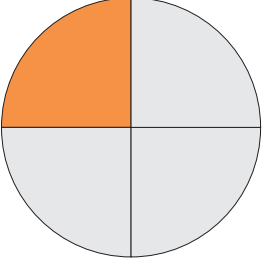
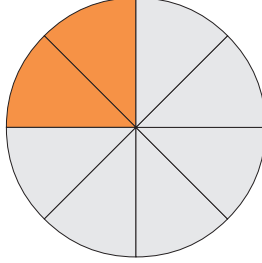
Worksheet

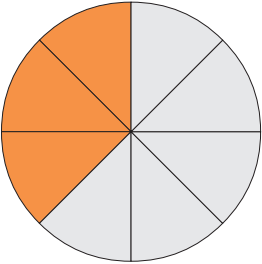
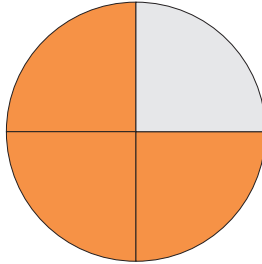
Identifying Equivalent Fractions

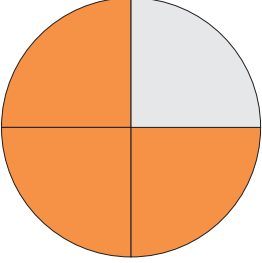
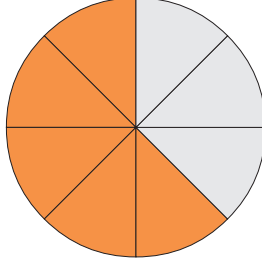
Objective: Determine whether two fractions are equivalent with pictures and numerals.

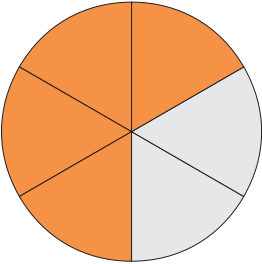
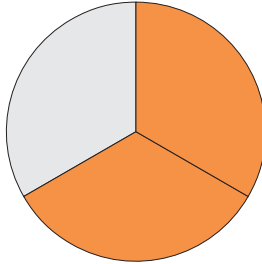
Directions: Label these fractions. Draw a line between the fractions that are equivalent.

	_____	_____	
		_____	
_____ is equivalent to _____.		_____	

	_____	_____	
_____ is equivalent to _____.			
_____ is equivalent to _____.			
_____ is equivalent to _____.			

	_____	_____	
_____ is equivalent to _____.			
_____ is equivalent to _____.			
_____ is equivalent to _____.			

	_____	_____	
_____ is equivalent to _____.			
_____ is equivalent to _____.			
_____ is equivalent to _____.			

	_____	_____	
_____ is equivalent to _____.			
_____ is equivalent to _____.			
_____ is equivalent to _____.			



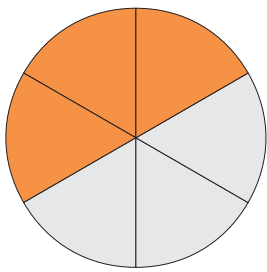
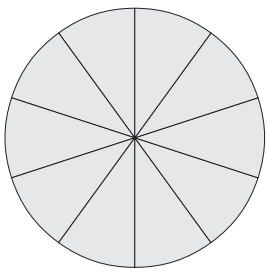
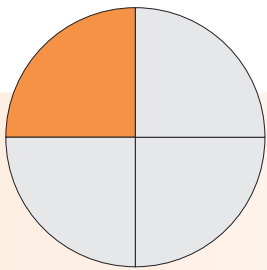
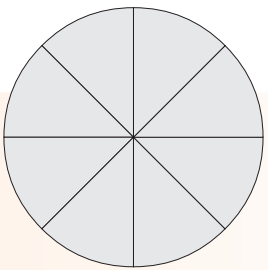
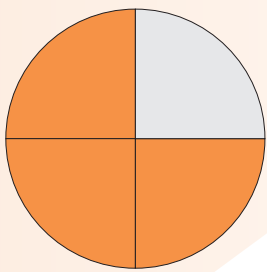
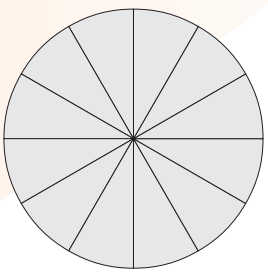
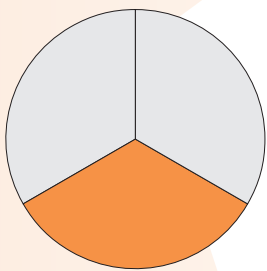
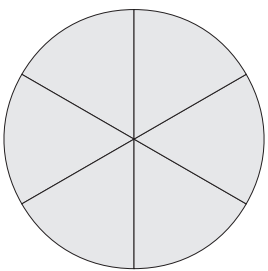
_____ is equivalent to _____.

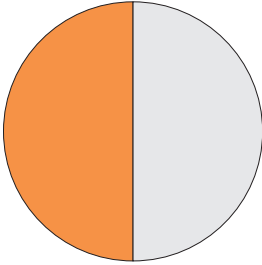
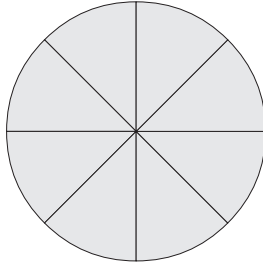
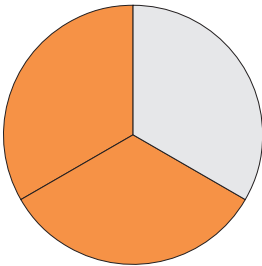
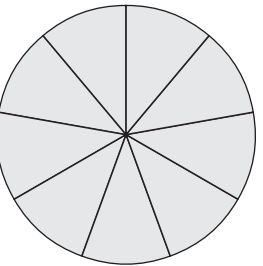
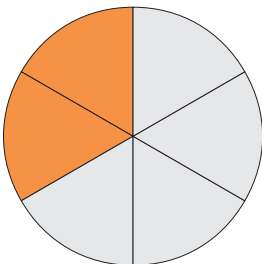
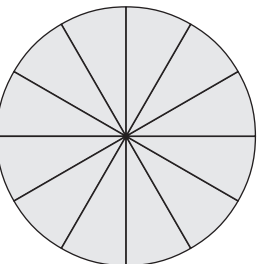
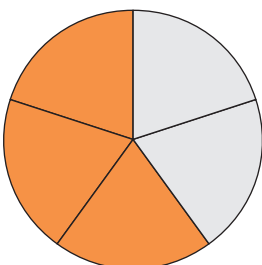
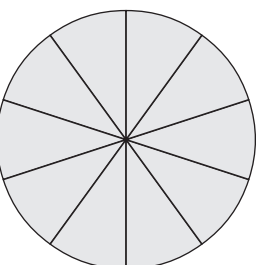
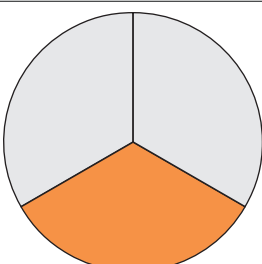
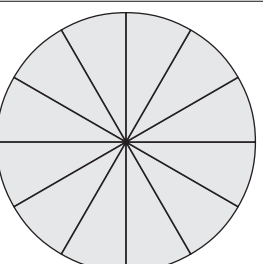
Worksheet

Making Equivalent Fractions

Objective: Given one fraction, generate an equivalent fraction using models.

Directions: Shade the blank fraction circle to make an equivalent fraction.
Name the fractions.

		_____ is equivalent to _____.
		_____ is equivalent to _____.
		_____ is equivalent to _____.
		_____ is equivalent to _____.

		_____ is equivalent to _____.
		_____ is equivalent to _____.
		_____ is equivalent to _____.
		_____ is equivalent to _____.
		_____ is equivalent to _____.

Worksheet

Finding Equivalent Fractions #1

Objective: Given one fraction, tell whether a second fraction is equivalent or not equivalent.

Directions:

1. Look at the two fractions.
2. Check to see whether they are equivalent.
3. Write “Equivalent” or “Not Equivalent” for each pair of fractions.

Fraction 1	Fraction 2	Show Your Work	Equivalent or Not Equivalent?
$\frac{1}{2}$	$\frac{2}{3}$		
$\frac{1}{4}$	$\frac{2}{8}$		
$\frac{2}{3}$	$\frac{6}{9}$		
$\frac{3}{4}$	$\frac{5}{12}$		

Fraction 1	Fraction 2	Show Your Work	Equivalent or Not Equivalent?
$\frac{2}{4}$	$\frac{6}{12}$		
$\frac{1}{6}$	$\frac{3}{8}$		
$\frac{1}{3}$	$\frac{2}{6}$		

Worksheet

Finding Equivalent Fractions #2

Objective: Given one fraction, identify an equivalent fraction without using pictures.

Directions:

1. Choose a fraction that you think is equivalent to the fraction in the first column.
2. Use the equivalent fraction rule to check whether the fractions are equivalent.
3. Repeat until you have found a pair of equivalent fractions for each problem.

Fraction	Pick the Equivalent Fraction	Show Your Work
$\frac{1}{2}$	$\frac{2}{3}$ $\frac{4}{5}$ $\frac{3}{8}$ $\frac{2}{4}$ $\frac{5}{8}$	_____ is equivalent to _____.
$\frac{1}{4}$	$\frac{2}{4}$ $\frac{1}{3}$ $\frac{2}{8}$ $\frac{5}{6}$ $\frac{7}{8}$	_____ is equivalent to _____.
$\frac{2}{3}$	$\frac{1}{5}$ $\frac{3}{4}$ $\frac{2}{6}$ $\frac{9}{10}$ $\frac{4}{6}$	_____ is equivalent to _____.
$\frac{3}{4}$	$\frac{9}{12}$ $\frac{3}{5}$ $\frac{6}{7}$ $\frac{1}{2}$ $\frac{5}{10}$	_____ is equivalent to _____.

Fraction	Pick the Equivalent Fraction	Show Your Work
$\frac{2}{4}$	$\frac{1}{3}$ $\frac{6}{12}$ $\frac{2}{9}$ $\frac{4}{10}$ $\frac{2}{6}$	_____ is equivalent to _____.
$\frac{1}{6}$	$\frac{4}{6}$ $\frac{3}{8}$ $\frac{2}{12}$ $\frac{1}{12}$ $\frac{4}{10}$	_____ is equivalent to _____.
$\frac{1}{3}$	$\frac{3}{4}$ $\frac{4}{8}$ $\frac{6}{9}$ $\frac{4}{12}$ $\frac{7}{12}$	_____ is equivalent to _____.

3. Fraction Magnitude

Sample Activities 1-2

- a. Activity One: Comparing Fractions With Different Denominators**
- b. Activity Two: Comparing Fractions With Different Denominators**

Worksheets

- a. Fraction Magnitude: Comparing Fractions With Different Denominators**
- b. Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators**

Sample Fraction Magnitude Activities (1–2)

College- and Career-Ready Standards:

4.NF.2. Compare two fractions with different numerators and different denominators, for example, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, for example, by using a visual fraction model.

Activity One: Comparing Fractions With Different Denominators

Purpose: To compare fraction magnitude between two fractions by finding common denominators.

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.

Materials (available for download from NCII):

Comparison flashcards (see Supplemental Materials section)

Multiplication chart (optional; see Supplemental Materials section)

Fraction tiles or fraction circles for justifying conclusions (see Supplemental Materials section)

Number line (optional; see Supplemental Materials section)

Worksheet: Fraction Magnitude: Comparing Fractions With Different Denominators (for extra practice)

Worksheet: Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators (for extra practice)

Modeling 1 (only one fraction is changed):

1. Present or write two fractions with different denominators ($\frac{4}{6}$ and $\frac{5}{12}$).
2. Point to the denominators (6 and 12) and say, “These are not the same.”
3. Explain that we need to change one or both of the fractions so the denominators are the same. When we rewrite a fraction, it must be equivalent.
4. Explain you should look at the smaller denominator first to see whether it is a factor of the larger denominator.
5. Explain 6 is a factor of 12: 6 times 2 equals 12. (If you are using a multiplication chart, show 6 times 2 equals 12 on the chart.)
6. Explain that to write an equivalent fraction, you multiply the numerator and the denominator by the same number.
7. Explain that we multiply $\frac{4}{6}$ times $\frac{2}{2}$ to rewrite $\frac{4}{6}$ as an equivalent fraction with 12 in the denominator.
8. Demonstrate setting up the multiplication.
9. Perform the multiplication to get $\frac{8}{12}$ as the answer.
10. Explain that now that $\frac{8}{12}$ and $\frac{5}{12}$ have the same denominator, it is time to compare!
11. Explain that when fractions have the same denominator, it is easy to compare. The fraction with the bigger numerator is the bigger fraction.
12. Place a greater-than sign between $\frac{8}{12}$ and $\frac{5}{12}$. (If the student does not remember which sign is which, remind him or her that the open part of the sign faces the bigger fraction.)
13. Read the answer: $\frac{8}{12}$ is greater than $\frac{5}{12}$.
14. Now let’s check it with the tiles or circles.
15. Demonstrate making $\frac{8}{12}$ and $\frac{5}{12}$ with either tiles or circles.
16. Explain that because $\frac{8}{12}$ is bigger than $\frac{5}{12}$, we know we are right!

Modeling 2 (both fractions are changed):

1. Present two fractions with different denominators ($\frac{1}{3}$ and $\frac{3}{4}$).
2. Point to the denominators (3 and 4) and say, “These are not the same.”
3. Explain we need to change one or both of the fractions so that the denominators are the same. When we rewrite a fraction, it *must* be equivalent.
4. Explain that you should look at the smaller denominator first to see whether it is a factor of the larger denominator.
5. Explain that 3 is *not* a factor of 4. You cannot divide 4 by 3 and get a whole number. (If you are using a multiplication chart, show the student that 3 is not a factor of 4.)
6. Explain that you need to write equivalent fractions for *both* fractions and have to decide on the least common denominator.
7. For each fraction, you will multiply the numerator and the denominator by the denominator of the other fraction. (See the example that follows.)

$$\frac{1}{3} \frac{(4)}{(4)} \quad \text{and} \quad \frac{3}{4} \frac{(3)}{(3)}$$

8. Explain that to write an equivalent fraction, you multiply the numerator and the denominator by the *same number*.
9. Demonstrate setting up the multiplication.
10. Explain that we multiply $\frac{1}{3}$ by $\frac{4}{4}$ to rewrite $\frac{1}{3}$ as an equivalent fraction. The new fraction is $\frac{4}{12}$.
11. Next, explain that we multiply $\frac{3}{4}$ by $\frac{3}{3}$ to rewrite $\frac{3}{4}$ as an equivalent fraction. The new fraction is $\frac{9}{12}$.
12. Both fractions are rewritten with 12 in the denominator: $\frac{4}{12}$ and $\frac{9}{12}$.
13. Explain that now that $\frac{4}{12}$ and $\frac{9}{12}$ have the same denominator, it is time to compare!
14. Explain that when fractions have the same denominators, it is easy to compare. The fraction with the bigger numerator is the bigger fraction.

15. Place a less-than sign between $\frac{4}{12}$ and $\frac{9}{12}$. (If the student does not remember which sign is which, remind him or her that the open part of the sign faces the bigger fraction.)
16. Read the answer: $\frac{4}{12}$ is less than $\frac{9}{12}$.
17. Demonstrate checking work by making $\frac{4}{12}$ and $\frac{9}{12}$ with either tiles or circles.
18. Explain that because $\frac{4}{12}$ is smaller than $\frac{9}{12}$, we know we are right!

Guided Practice:

1. Present or write two fractions with different denominators.
2. Ask the student to look at the denominator. Are they the same?
3. The student decides the denominators are not the same.
4. Direct the student to see whether the smaller denominator is a multiple of the larger denominator.
5. If yes, the student multiplies the numerator and the denominator of the fraction with the smaller denominator by the factor to make the denominators the same.
6. If no, the student multiplies the numerator and the denominator of each fraction with the denominator of the other fraction. (See the sample under Modeling 2.)
7. The student finds the new equivalent fraction(s) so that both fractions have the same denominator.
8. It's time to compare.
9. The student determines which fraction is bigger.
10. The student writes the $<$, $>$, or $=$ sign between the fractions.
11. The student checks the work with tiles or circles.

Corrective Feedback:

Incorrect student response 1: The student cannot determine whether the smaller denominator is a factor of the larger denominator.

Teacher feedback:

Option 1: The teacher should multiply the smaller denominator (review the multiplication facts) by 1, 2, 3, and so on, until the other denominator is an answer (showing it is a multiple) or until the other denominator gets surpassed (showing it is not a multiple). The student should practice finding the correct response.

Option 2: The teacher should use the multiplication chart to show the multiples to determine whether it is a multiple of the larger denominator. The student should practice finding the correct response.

Incorrect student response 2: The student cannot recall multiplication facts when writing the new equivalent fraction.

Teacher feedback: Use the multiplication chart as an aid as needed.

Incorrect student response 3: The student cannot distinguish between greater than and less than signs.

Teacher feedback: Ask the student to pretend the sign is an alligator (something that likes to eat and has a big mouth). The open mouth always wants to eat the bigger amount. Provide a few examples for practice.

Always have the student demonstrate the correct response before moving on.



Activity Two: Comparing Fractions With Different Denominators

Purpose: To compare fraction magnitude between two fractions by using benchmark fractions (numbers and fractions used for benchmarks: 0, $\frac{1}{2}$, 1).

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.

Materials (available for download from NCII):

Fraction Comparison flashcards (see Supplemental Materials section)

Fraction tiles or fraction circles—for justifying conclusions (see Supplemental Materials section)

Number lines with various denominators marked (see Supplemental Materials section)

Worksheet: Fraction Magnitude: Comparing Fractions With Different Denominators

Worksheet: Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators

Modeling:

1. Present or write two fractions with different denominators ($\frac{4}{6}$ and $\frac{5}{12}$).

**Sample
Teacher Talk:**

2. Point to the denominators (6 and 12) and say, “These are not the same.”

3. Explain that we need to think about the size of each fraction to decide which is bigger.

4. Explain that you look at the first fraction, $\frac{4}{6}$, and think “Is it close to 0?”

5. I decide it is not close to 0. I think, “Is it close to $\frac{1}{2}$?”
6. I decide it is a little bigger than $\frac{1}{2}$ because $\frac{3}{6}$ is equal to $\frac{1}{2}$.
7. Say, “Now I look at the next fraction, $\frac{5}{12}$, and think ‘Is it close to 0?’”
8. I decide it is not close to 0. I think, “Is it close to $\frac{1}{2}$?”
9. I decide it is a little smaller than $\frac{1}{2}$ because $\frac{6}{12}$ is equal to $\frac{1}{2}$.
10. Explain that both fractions are close to $\frac{1}{2}$; $\frac{4}{6}$ is a little bigger than $\frac{1}{2}$, and $\frac{5}{12}$ is a little smaller than $\frac{1}{2}$. Now I know $\frac{4}{6}$ is bigger.
11. Place a greater-than sign between $\frac{4}{6}$ and $\frac{5}{12}$. (If the student does not remember which sign is which, remind him or her that the open part of the sign faces the bigger fraction.)
12. Read the answer: $\frac{4}{6}$ is greater than $\frac{5}{12}$.
13. Now let’s check it with the tiles and circles.
14. Demonstrate making $\frac{4}{6}$ and $\frac{5}{12}$ with either tiles or circles.
15. Explain that because $\frac{4}{6}$ is bigger than $\frac{5}{12}$, we know we are right!

Guided Practice:

1. Present or write two fractions with different denominators.
2. Ask the student to look at the denominators. Are they the same?
3. The student decides the denominators are not the same.
4. Direct the student to compare the first fraction to 0, $\frac{1}{2}$, and 1.
5. The student should think about whether the fraction is a little bigger than each of these benchmark numbers.
6. After determining the relationship of the two new fractions to the benchmark numbers, the student should reason about each fraction’s magnitude.
7. The student determines which fraction is bigger.
8. The student writes the $<$, $>$, or $=$ sign between the fractions.
9. The student checks the work with tiles or circles.

Corrective Feedback:

Sample Incorrect student response 1: The student cannot determine whether one of the target fractions is close to one of the three benchmark numbers.

Teacher feedback:

Option 1: The teacher can present a number line to help the student visualize where 0, $\frac{1}{2}$, and 1 go on the number line. The teacher helps the student place one of the target fractions on the line to see whether the fraction is close to 0, close to 1, or less than or greater than $\frac{1}{2}$. The teacher prompts the student to then reason about the fraction magnitude and then aids in comparing the two target fractions. The student demonstrates correct procedures before moving to the next set of fractions.

Option 2: The teacher can use tiles or circles to show the target fraction and compare it to $\frac{1}{2}$ and 1. The visualization should help the student reason about the fraction magnitude and then aid in comparing the two target fractions. The student demonstrates correct procedures before moving to the next set of fractions.

Sample incorrect student response 2: The student cannot distinguish between greater-than and less-than signs.

Teacher feedback: Ask the student to pretend the sign is an alligator (something that likes to eat and has a big mouth). The open mouth always wants to eat the bigger amount.

(Have the student demonstrate the correct response before moving on.)

Worksheet

Fraction Magnitude: Comparing Fractions With Different Denominators

Objective: Given two fractions, compare them using greater than (>), less than (<), or equal to (=). Write the fractions with a common denominator when necessary.

Note: If the student struggles with this worksheet, try the Scaffolded Fraction Magnitude Worksheet.

Directions:

1. Look at the two fractions. Can you compare them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Compare the fractions using the greater than (>), less than (<), or equal to (=) symbol.
4. Check your work with the fraction tiles.
 - a. If your answer is correct, draw a check mark and move to the next problem.
 - b. If your answer is incorrect, go back and fix your work.

Fraction 1	Fraction 2	Show Work	<, >, or =	Check With Tiles ✓
$\frac{1}{2}$	$\frac{3}{6}$			
$\frac{1}{4}$	$\frac{3}{8}$			

Fraction 1	Fraction 2	Show Work	<, >, or =	Check With Tiles ✓
$\frac{2}{9}$	$\frac{6}{9}$			
$\frac{3}{4}$	$\frac{1}{12}$			
$\frac{2}{4}$	$\frac{1}{5}$			
$\frac{1}{6}$	$\frac{2}{3}$			

Worksheet

Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators

Note: You may choose to use this worksheet prior to the Fraction Magnitude Activity, to make sure that the student has achieved mastery of each step before moving to the next.

Objective: Given two fractions, compare them using greater than (>), less than (<), or equal to (=). Write the fractions with a common denominator when necessary.

Directions:

1. Look at the two fractions. Are the denominators equal?

Fraction 1	Fraction 2	Are the Denominators Equal? Yes or No
$\frac{1}{2}$	$\frac{3}{8}$	
$\frac{2}{7}$	$\frac{6}{7}$	
$\frac{1}{4}$	$\frac{3}{4}$	
$\frac{2}{5}$	$\frac{5}{6}$	
$\frac{3}{4}$	$\frac{1}{3}$	
$\frac{5}{8}$	$\frac{6}{8}$	
$\frac{3}{5}$	$\frac{5}{10}$	

Directions:

1. Look at the two fractions. Can you compare them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Check your work with the fraction tiles.
 - a. If your answer is correct, draw a check mark and move to the next problem.
 - b. If your answer is incorrect, go back and check your work.

Fraction 1	Fraction 2	Are the Denominators Equal? Yes or No	Show Work and Write the Common Denominator
$\frac{3}{9}$	$\frac{1}{3}$		
$\frac{1}{4}$	$\frac{5}{12}$		
$\frac{2}{7}$	$\frac{3}{7}$		
$\frac{1}{2}$	$\frac{4}{5}$		

Directions:

1. Look at the two fractions. Can you compare them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Compare the fractions using the greater than (>), less than (<), or equal to (=) symbol.
4. Check your work with the fraction tiles.
 - a. If your answer is correct, draw a check mark and move to the next problem.
 - b. If your answer is incorrect, go back and check your work.

Fraction 1	Fraction 2	Show Work and Write the Common Denominator	Compare: <, >, or =
$\frac{2}{5}$	$\frac{3}{4}$		
$\frac{2}{3}$	$\frac{1}{3}$		
$\frac{5}{7}$	$\frac{1}{2}$		
$\frac{1}{4}$	$\frac{3}{8}$		
$\frac{4}{5}$	$\frac{7}{10}$		

4. Converting Between Mixed Numbers and Improper Fractions

Sample Activity

a. Converting Mixed Numbers and Improper Fractions

Worksheet

a. Converting Mixed Numbers and Improper Fractions

Sample Activity: Converting Mixed Numbers and Improper Fractions

College- and Career-Ready Standards:

This activity does not directly correlate to one specific domain; however, it is relevant for understanding fractions greater than 1 and the ways to represent them. See the following related standards.

4.NF. Overall statement on Fractions: Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

- Understand a fraction a/b with $a > 1$ as a sum of a fractions $1/b$.
 - c. Add and subtract mixed numbers with like denominators, for example, by replacing each mixed number with an equivalent fraction, or by using properties of operations and the relationship between addition and subtraction.

5.NF. Use equivalent fractions as a strategy to add and subtract fractions.

- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

Activity: Using Fraction Tiles or Fraction Circles; Showing Mixed Numbers Equivalent to Improper Fractions

Purpose: Understand improper fractions and their mixed-number equivalents.

Materials (available for download from NCII):

- Improper Fraction and Mixed Number Flash Cards (see Supplemental Materials section)

- Fraction circles (see Supplemental Materials section; print two copies)
- **Worksheet:** Understanding and Converting Mixed Numbers and Improper Fractions

Prerequisite Vocabulary:

Equivalent, numerator, denominator, improper fraction, mixed number

Modeling (improper fractions to mixed numbers):

1. The teacher shows $5/4$ and explains that this is an improper fraction.
2. The teacher explains that this fraction is improper because it is greater than 1.
3. The fraction is greater than 1 because the numerator (5) is greater than the denominator (4).
4. The teacher explains that improper fractions have a mixed number that is equivalent.
5. The teacher reminds the student that improper fractions and mixed numbers are always greater than 1.
6. The teacher explains that $5/4$ is the same as five $1/4$ pieces.
7. The teacher demonstrates what $5/4$ looks like with fraction circles.
8. The teacher explains that he or she chooses the $1/4$ pieces and counts 5 of them (must be using two sets to create fractions greater than 1).
9. The five $1/4$ pieces are now on the table.
10. The teacher puts four of the $1/4$ pieces together to make 1 whole.
11. The teacher places the other $1/4$ piece next to the whole that was created with $4/4$.
12. The teacher explains that $5/4$ is the same as one whole and $1/4$. This means $5/4 = 1$ and $1/4$.

Modeling (mixed numbers to improper fractions):

1. The teacher shows 1 and $2/5$ and explains that it is a mixed number.
2. The teacher explains that the mixed number is greater than 1 because it is 1 and $2/5$. It has a whole number and a proper fraction.

3. The teacher explains that mixed numbers have an improper fraction that is equivalent.
4. The teacher reminds the student that mixed numbers and improper fractions are always greater than 1.
5. The teacher demonstrates what 1 and $\frac{2}{5}$ looks like with fraction circles.
6. The teacher explains that he or she chooses the one whole and two of the $\frac{1}{5}$ pieces.
7. The teacher counts two of the $\frac{1}{5}$ pieces. (The teacher should have an additional packet of $\frac{1}{5}$ pieces handy to show the equivalence in Steps 12–13).
8. The one whole and two $\frac{1}{5}$ pieces are now on the table.
9. The teacher puts the two $\frac{1}{5}$ pieces together to make $\frac{2}{5}$.
10. The teacher explains that this shows 1 and $\frac{2}{5}$.
11. The teacher explains that to find the improper fraction equivalent to 1 and $\frac{2}{5}$, we need to put enough fifths together to make one whole.
12. On top of the one whole (to fill in the region), the teacher counts five of the $\frac{1}{5}$ pieces.
13. The teacher explains that $\frac{5}{5}$ equals one whole.
14. To find the improper fraction, the teacher explains he or she counts all the $\frac{1}{5}$ pieces.
15. The teacher counts 1, 2, 3, 4, 5, 6, 7.
16. The teacher explains that $\frac{7}{5}$ is the same as one whole and $\frac{2}{5}$. This means 1 and $\frac{2}{5} = \frac{7}{5}$.

Guided Practice (improper fractions to mixed numbers):

1. The teacher shows $\frac{4}{3}$ and the student determines whether the task is to change an improper fraction to a mixed number or a mixed number to an improper fraction.
2. The student states this is an improper fraction and he or she will find the mixed-number equivalent.
3. The student explains this fraction is improper because it is greater than 1.

4. The fraction is greater than 1 because the numerator (4) is greater than the denominator (3).
5. The student explains that $\frac{4}{3}$ is the same as four $\frac{1}{3}$ pieces.
6. The student demonstrates what $\frac{4}{3}$ looks like with fraction circles.
7. The student explains that he or she chooses the $\frac{1}{3}$ pieces and counts four of them (must be using two sets to create fractions greater than 1).
8. The four $\frac{1}{3}$ pieces are now on the table.
9. The student puts three of the $\frac{1}{3}$ pieces together to make one whole.
10. The student places the other $\frac{1}{3}$ piece next to the whole that was created with $\frac{3}{3}$.
11. The student explains that $\frac{4}{3}$ is the same as one whole and $\frac{1}{3}$. This means $\frac{4}{3} = 1$ and $\frac{1}{3}$.

Guided Practice (mixed numbers to improper fractions):

1. The student determines whether the task is to change an improper fraction to a mixed number or a mixed number to an improper fraction.
2. The student shows 1 and $\frac{3}{4}$ and explains that this is a mixed number.
3. The student explains that the mixed number is greater than 1 because it is 1 and $\frac{3}{4}$. It has a whole number and a proper fraction.
4. The student demonstrates what 1 and $\frac{3}{4}$ looks like with fraction circles.
5. The student explains that he or she chooses the one whole and three of the $\frac{1}{4}$ pieces.
6. The student counts three of the $\frac{1}{4}$ pieces. (The teacher should have an additional packet of $\frac{1}{4}$ pieces handy for the student to show the equivalence in Steps 10–12).
7. The one whole and three of the $\frac{1}{4}$ pieces are now on the table.
8. The student puts the three $\frac{1}{4}$ pieces together to make $\frac{3}{4}$.
9. The student explains that this shows 1 and $\frac{3}{4}$.
10. The student explains that to find the improper fraction equivalent to 1 and $\frac{3}{4}$, we need to put enough fourths together to make one whole.

11. On top of the one whole (to fill in the region), the student counts four of the $\frac{1}{4}$ pieces.
12. The student explains that $\frac{4}{4}$ equals one whole.
13. To find the improper fraction, the student explains that he or she counts all the $\frac{1}{4}$ pieces.
14. The student counts 1, 2, 3, 4, 5, 6, 7.
15. The student explains that $\frac{7}{4}$ is the same as one whole and $\frac{3}{4}$. This means 1 and $\frac{3}{4} = \frac{7}{4}$.

Corrective Feedback:

Sample incorrect student response 1: The student has difficulty articulating, following, or anticipating the steps.

Teacher feedback:

Option 1: The teacher should help with the explanation. This concept is difficult when first taught, and much practice will be needed before the student will be able to model the activity as completely as the teacher.

Option 2: The teacher could consider scaffolding the steps less dramatically than as shown in earlier sections of this activity and ask the student questions during the process to get him or her thinking and talking about the fractions. Then after a lot of practice, reduce prompts or additional help from the teacher.

Sample incorrect student response 2: The student has difficulty distinguishing mixed numbers from improper fractions.

Teacher feedback: The teacher should provide the rules: (1) Mixed numbers *always* have a whole number with a fraction. (2) Improper fractions have a numerator that is greater than the denominator.

Sample incorrect student response 3: The student has difficulty finding the correct size of fraction pieces.

Teacher feedback: The teacher should direct the student to look at the denominator in the problem and choose the pieces with the same denominator.

Always have the student demonstrate the correct response before moving on.

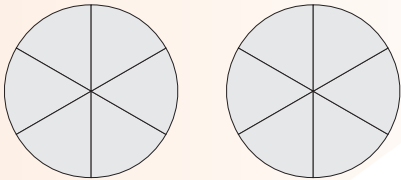
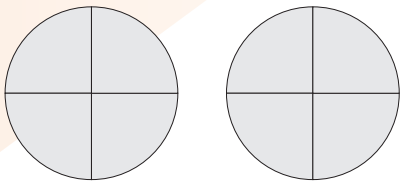
Worksheet

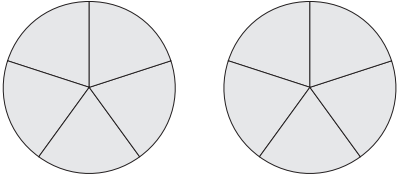
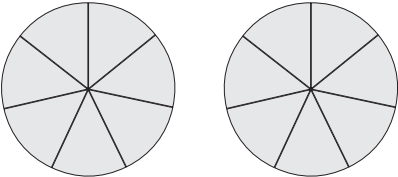
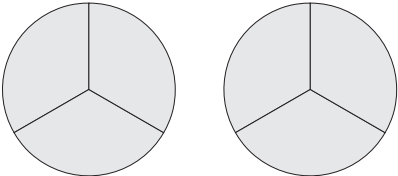
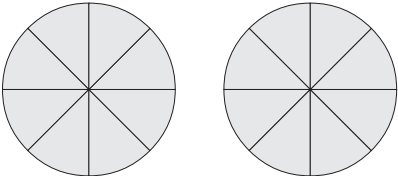
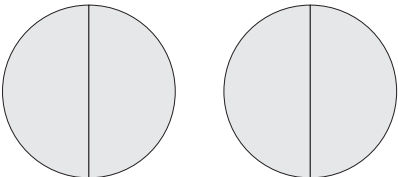
Converting Mixed Numbers and Improper Fractions

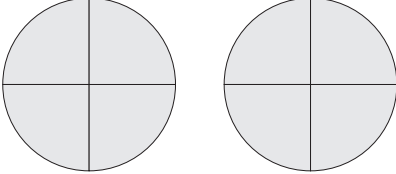
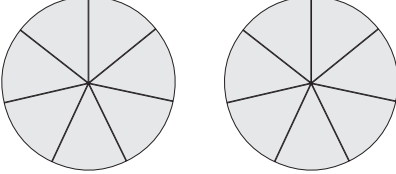
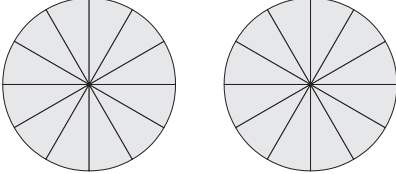
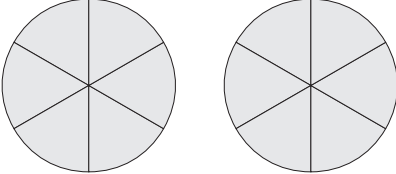
Objective: Determine whether a fraction is an improper fraction or a mixed number. Use fraction circles to represent and convert between improper fractions and mixed numbers.

Steps:

1. Look at the fraction. Is it an improper fraction or a mixed number?
2. Write the fraction in the correct column (either improper fraction or mixed number).
3. Use fraction circles to help convert the improper fraction or mixed number to its equivalent form.

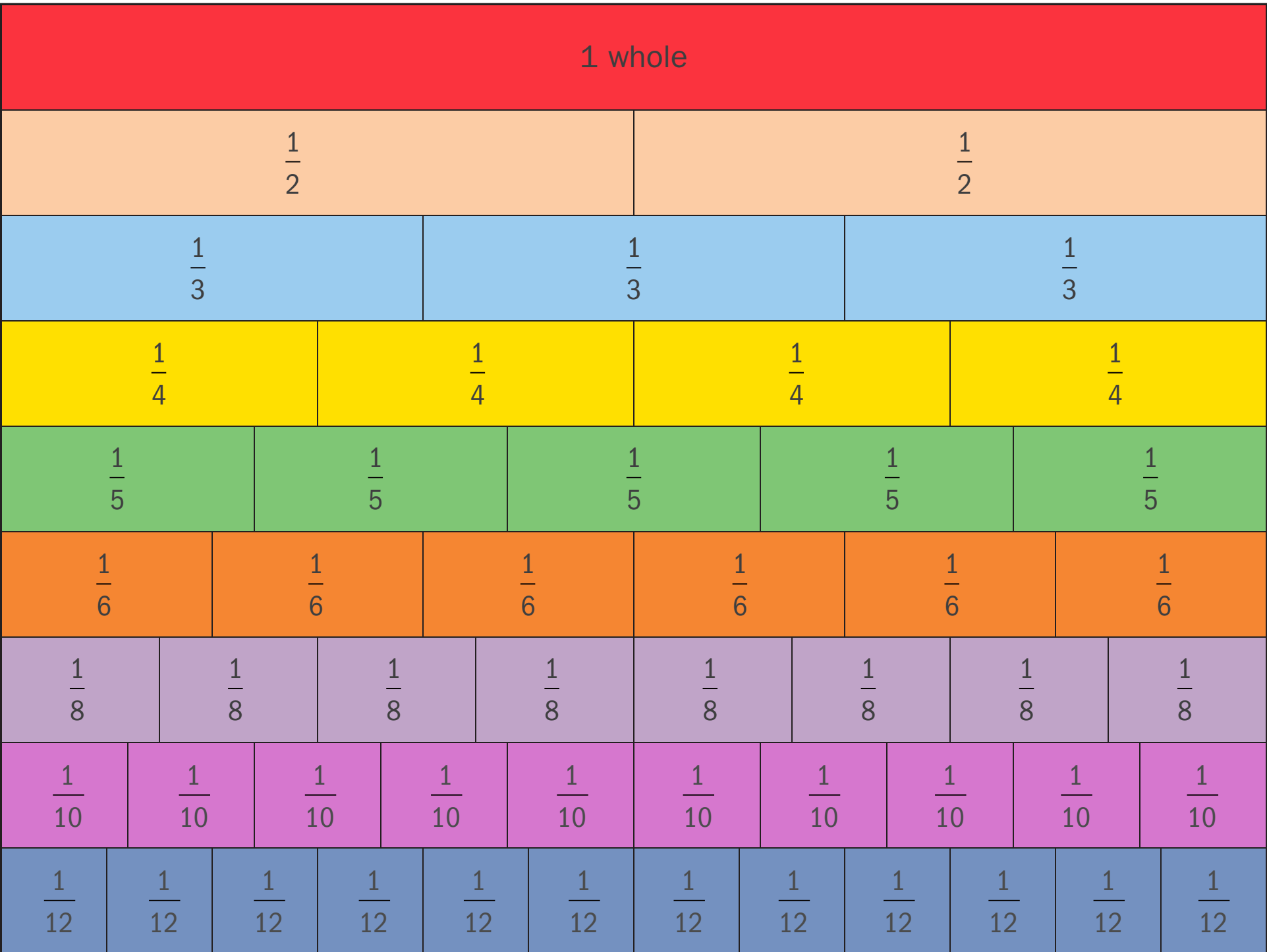
Fraction	Fraction Circles	Improper Fraction Example: $\frac{3}{2}$	Mixed Number Example: $1\frac{2}{5}$
$\frac{9}{6}$			
$1\frac{3}{4}$			

Fraction	Fraction Circles	Improper Fraction Example: $\frac{3}{2}$	Mixed Number Example: $1\frac{2}{5}$
$\frac{8}{5}$			
$\frac{11}{7}$			
$1\frac{2}{3}$			
$1\frac{5}{8}$			
$\frac{3}{2}$			

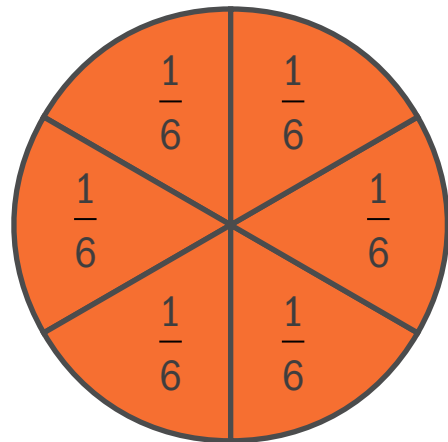
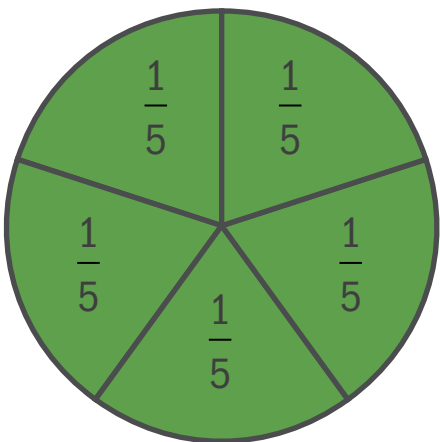
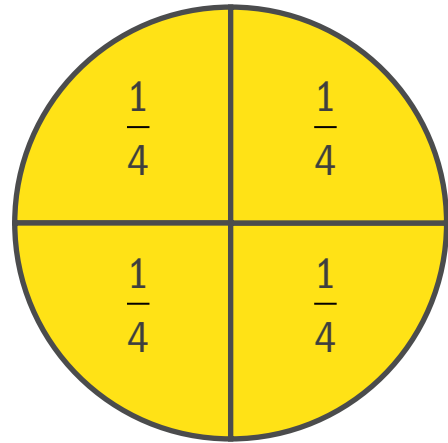
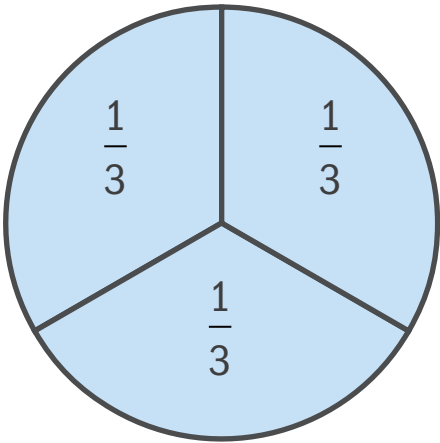
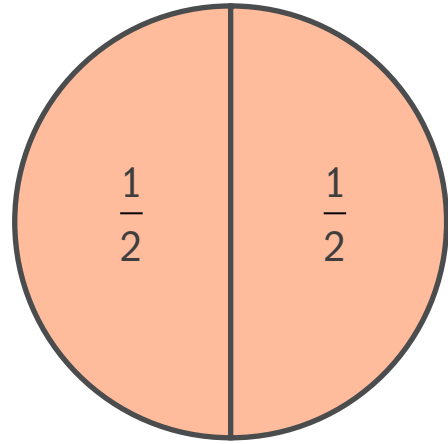
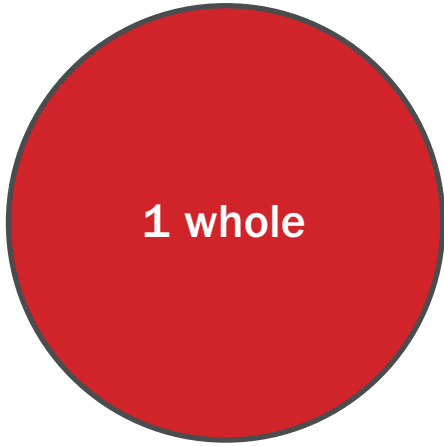
Fraction	Fraction Circles	Improper Fraction Example: $\frac{3}{2}$	Mixed Number Example: $1\frac{2}{5}$
$1\frac{3}{4}$			
$\frac{12}{7}$			
$1\frac{1}{12}$			
$\frac{10}{6}$			

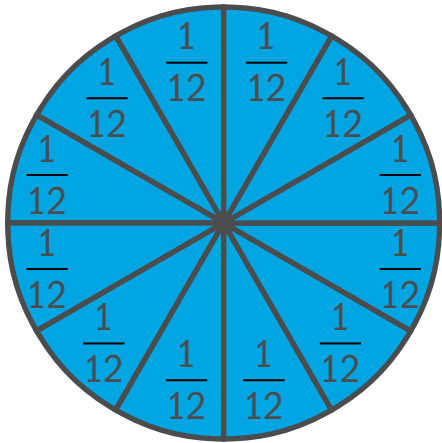
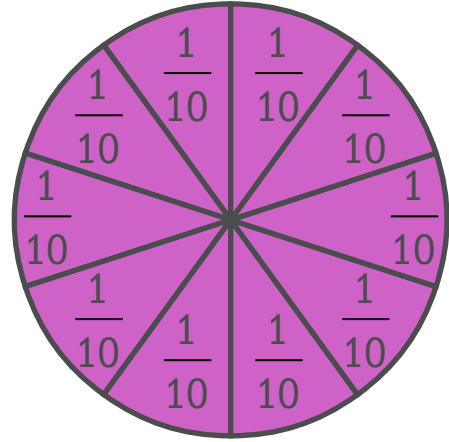
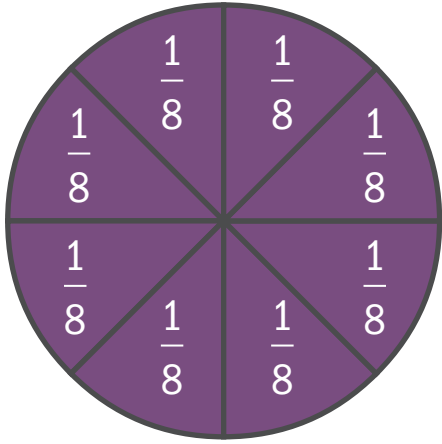
5. Supplemental Materials (used across activities)

- a. Fractions Tiles**
- b. Fraction Circles**
- c. Fraction Equivalence Cards With Images**
- d. Fraction Equivalence Cards Without Images**
- e. Best Time Score Card**
- f. Fraction Comparison Flash Cards**
- g. Multiplication Chart**
- h. Improper Fraction Cards**
- i. Mixed-Number Cards**
- j. Number Line**

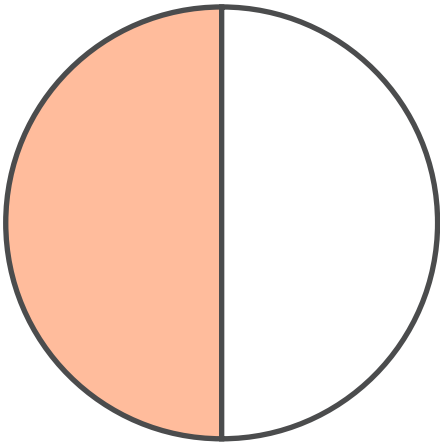


(Cut into pieces to use with sample activities.)

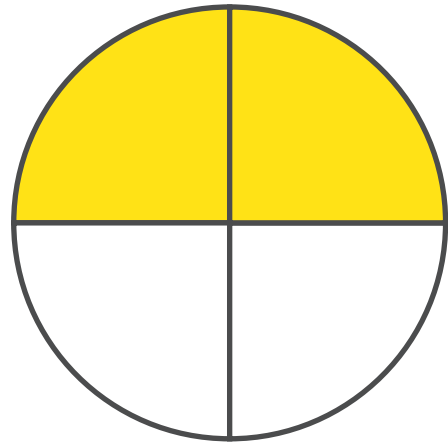




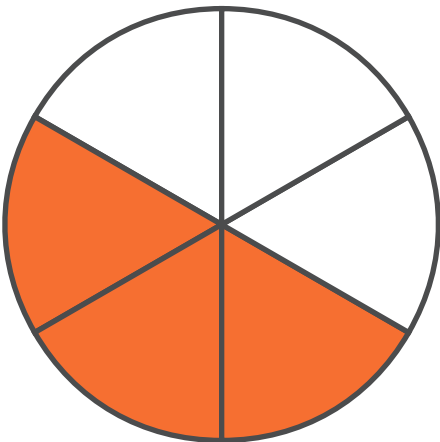
$$\frac{1}{2}$$



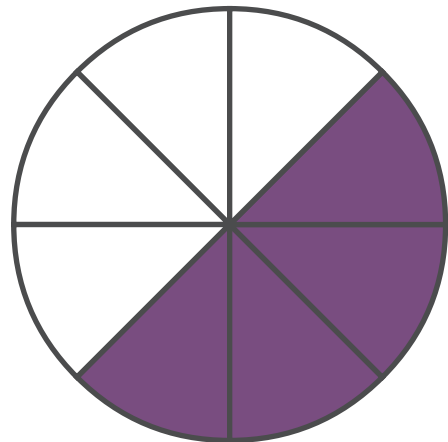
$$\frac{2}{4}$$



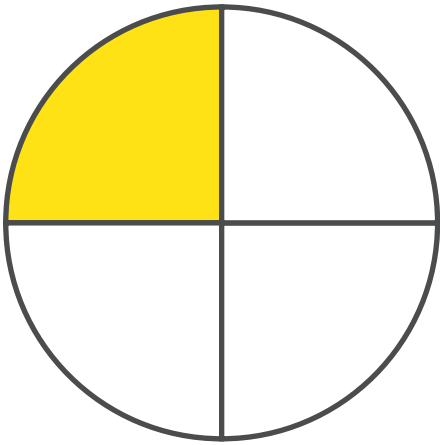
$$\frac{3}{6}$$



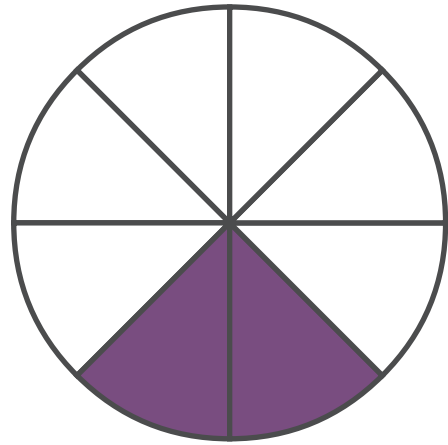
$$\frac{4}{8}$$



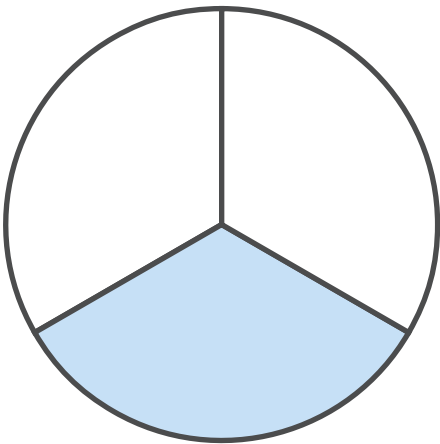
$$\frac{1}{4}$$



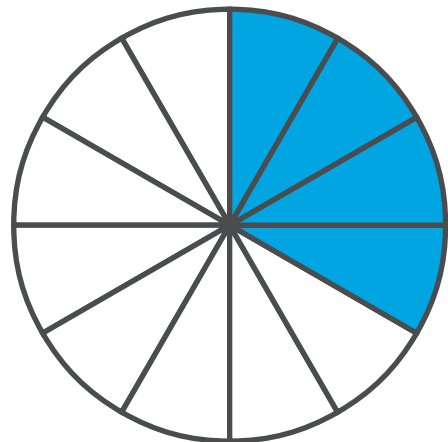
$$\frac{2}{8}$$



$$\frac{1}{3}$$



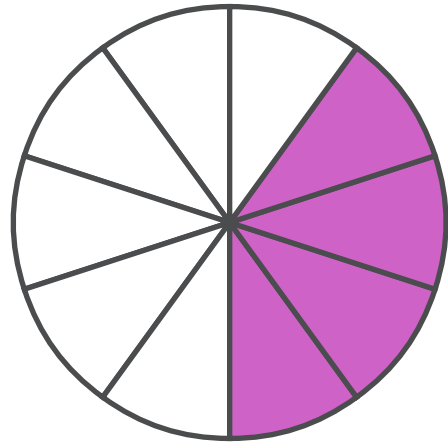
$$\frac{4}{12}$$



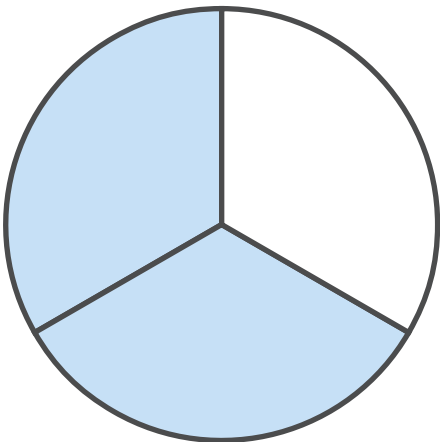
$$\frac{2}{5}$$



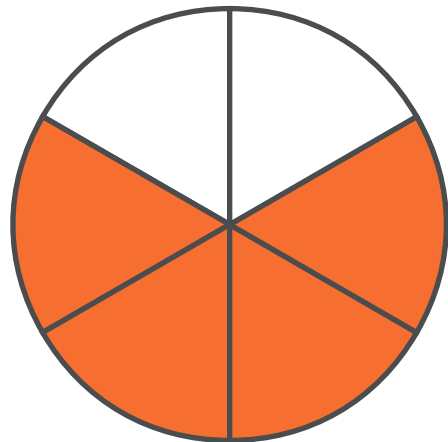
$$\frac{4}{10}$$



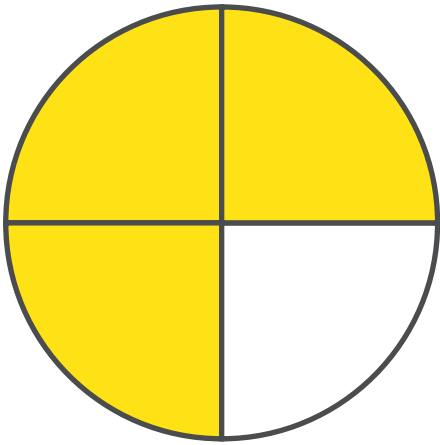
$$\frac{2}{3}$$



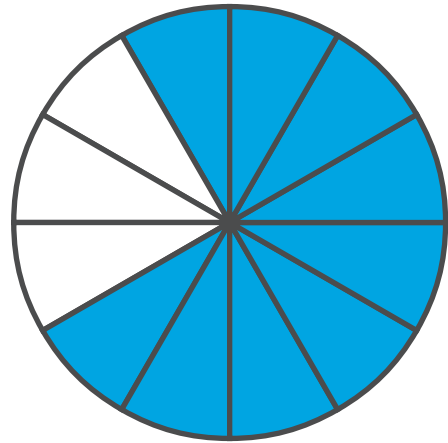
$$\frac{4}{6}$$



$$\frac{3}{4}$$



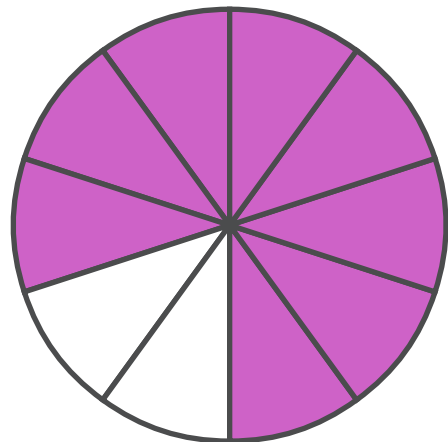
$$\frac{9}{12}$$



$$\frac{4}{5}$$



$$\frac{8}{10}$$



$$\frac{1}{2}$$

$$\frac{2}{4}$$

$$\frac{3}{6}$$

$$\frac{4}{8}$$

$$\frac{1}{4}$$

$$\frac{2}{8}$$

$$\frac{1}{3}$$

$$\frac{4}{12}$$

$$\frac{2}{5}$$

$$\frac{4}{10}$$

$$\frac{2}{3}$$

$$\frac{4}{6}$$

$$\frac{3}{4}$$

$$\frac{9}{12}$$

$$\frac{4}{5}$$

$$\frac{8}{10}$$

$$\frac{1}{5}$$

$$\frac{2}{10}$$

$$\frac{2}{6}$$

$$\frac{3}{9}$$

$$\frac{2}{3}$$

$$\frac{6}{9}$$

$$\frac{1}{4}$$

$$\frac{3}{12}$$

National Center on
INTENSIVE INTERVENTION

at American Institutes for Research ■

Student Name:

	Score	Score	Score	Best Score
Date:				
Date:				
Date:				
Date:				
Date:				

Student Name:

	Score	Score	Score	Best Score
Date:				
Date:				
Date:				
Date:				
Date:				

Student Name:

	Score	Score	Score	Best Score
Date:				
Date:				
Date:				
Date:				
Date:				

$$\frac{4}{6} \square \frac{5}{12}$$

$$\frac{3}{4} \square \frac{1}{2}$$

$$\frac{1}{4} \square \frac{2}{8}$$

$$\frac{5}{10} \square \frac{3}{5}$$

$$\frac{2}{6} \square \frac{1}{2}$$

$$\frac{2}{3} \square \frac{6}{9}$$

$$\frac{3}{4} \square \frac{8}{12}$$

$$\frac{1}{6} \square \frac{1}{3}$$

$$\frac{2}{5} \square \frac{6}{10}$$

$$\frac{1}{2} \square \frac{3}{4}$$

$$\frac{4}{9} \square \frac{2}{3}$$

$$\frac{3}{6} \square \frac{1}{3}$$

$$\frac{1}{5} \square \frac{3}{10}$$

$$\frac{1}{2} \square \frac{3}{8}$$

$$\frac{1}{3} \square \frac{3}{12}$$

$$\frac{2}{5} \square \frac{6}{15}$$

$$\frac{1}{2} \square \frac{2}{5}$$

$$\frac{1}{2} \square \frac{1}{3}$$

$$\frac{3}{5} \square \frac{3}{4}$$

$$\frac{2}{5} \square \frac{1}{3}$$

$$\frac{4}{9} \square \frac{1}{4}$$

$$\frac{2}{3} \square \frac{7}{8}$$

$$\frac{3}{10} \square \frac{3}{4}$$

$$\frac{2}{5} \square \frac{3}{5}$$

$$\frac{2}{6} \square \frac{3}{5}$$

$$\frac{3}{7} \square \frac{2}{4}$$

$$\frac{3}{9} \square \frac{2}{7}$$

$$\frac{5}{12} \square \frac{3}{8}$$

$$\frac{1}{2} \square \frac{3}{4}$$

$$\frac{4}{6} \square \frac{7}{11}$$

$$\frac{2}{7} \square \frac{1}{3}$$

$$\frac{5}{8} \square \frac{6}{10}$$

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

$$\frac{5}{4}$$

$$\frac{3}{1}$$

$$\frac{4}{1}$$

$$\frac{6}{1}$$

$$\frac{9}{1}$$

$$\frac{13}{1}$$

$$\frac{20}{1}$$

$$\frac{2}{1}$$

$$\frac{3}{2}$$

$$\frac{4}{2}$$

$$\frac{5}{2}$$

$$\frac{7}{2}$$

$$\frac{4}{3}$$

$$\frac{6}{3}$$

$$\frac{7}{3}$$

$$\frac{15}{3}$$

$$\frac{5}{4}$$

$$\frac{7}{4}$$

$$\frac{11}{4}$$

$$\frac{16}{4}$$

$$\frac{6}{5}$$

$$\frac{7}{5}$$

$$\frac{10}{5}$$

$$\frac{12}{5}$$

$$\frac{7}{6}$$

$$\frac{10}{6}$$

$$\frac{11}{6}$$

$$\frac{18}{6}$$

$$\frac{8}{7}$$

$$\frac{10}{7}$$

$$\frac{14}{7}$$

$$\frac{17}{7}$$

$$\frac{9}{8}$$

$$\frac{10}{8}$$

$$\frac{16}{8}$$

$$\frac{19}{8}$$

$$\frac{11}{10}$$

$$\frac{13}{10}$$

$$\frac{16}{10}$$

$$\frac{20}{10}$$

13

—

12

15

—

12

20

—

12

24

—

12

$$1\frac{1}{2}$$

$$2\frac{1}{2}$$

$$3\frac{1}{2}$$

$$4\frac{1}{2}$$

$$1\frac{2}{3}$$

$$1\frac{1}{3}$$

$$2\frac{2}{3}$$

$$3\frac{1}{3}$$

$$1 \frac{1}{3}$$

$$1 \frac{2}{3}$$

$$2 \frac{2}{3}$$

$$4 \frac{1}{3}$$

$$1 \frac{3}{4}$$

$$1 \frac{1}{4}$$

$$1 \frac{2}{4}$$

$$3 \frac{1}{4}$$

$$1 \frac{1}{5}$$

$$1 \frac{4}{5}$$

$$1 \frac{3}{5}$$

$$4 \frac{1}{5}$$

$$1 \frac{1}{6}$$

$$1 \frac{4}{6}$$

$$1 \frac{5}{6}$$

$$2 \frac{3}{6}$$

$$1 \frac{1}{7}$$

$$1 \frac{4}{7}$$

$$1 \frac{5}{7}$$

$$3 \frac{2}{7}$$

$$1 \frac{1}{8}$$

$$1 \frac{3}{8}$$

$$1 \frac{7}{8}$$

$$2 \frac{3}{8}$$

$$5 \frac{2}{3}$$

$$4 \frac{3}{4}$$

$$3 \frac{2}{5}$$

$$6 \frac{2}{6}$$

$$2 \frac{6}{7}$$

$$3 \frac{6}{8}$$

$$2 \frac{1}{9}$$

$$3 \frac{3}{10}$$

